

Decay of charged scalar field around a black hole: quasinormal modes of R-N and R-N-AdS black hole

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Abstract

It is well known that the charged scalar perturbations of the Reissner-Nordstrom metric will decay slower at very late times than the neutral ones, thereby dominating in the late time signal. We show that at the stage of quasinormal ringing, on the contrary, the neutral perturbations will decay slower both for RN and RNAdS black holes. The QN frequencies of the nearly extreme RN black hole have the same imaginary parts (damping times) for charged and neutral perturbations. The possible connection of the quasinormal spectrum behaviour near the extremal limit with the Choptuik scaling is discussed.

1 Introduction

The response of a Schwarzschild black hole as a Gaussian wave packet impinges upon it consists of decaying quasinormal oscillations, dominating after time $t \approx 70M$, and inverse power-low tails, dominating after time $t \approx 300M$, where M is the black hole mass (see [1] and references therein). The quasinormal ringing can be caused by either external fields or by the formation of a black hole itself, and, the characteristic frequencies do not depend on a form of perturbations, giving us a "footprint" of a black hole.

Due to AdS/CFT correspondence [2] the investigation of the quasinormal frequencies of AdS black holes is appealing now: it gives the thermalization time scale for a field perturbation [3], namely, the imaginary part of the quasinormal frequency, being inversely proportional to the damping time of a given mode, determines the relaxation time of a field. Thus the more imaginary part of ω the faster a given field comes to an equilibrium. The QN modes, corresponding to fields of different spin, have been obtained recently on the conformal field theory side for the three-dimensional BTZ black hole [4]. On the AdS gravity side, it was found that for the neutral massless scalar field in the background of RNAdS black hole with small charge, the more the black hole charge is, the quicker for its approach to thermal equilibrium in CFT [5], and after the black hole charge approaches some critical value the situation changes on contrary [6]. This repeats the behaviour of the usual R-N quasinormal spectrum, where the imaginary part of ω grows with the black hole charge up to some maximum, and then begin to decrease.

Summarizing the results of the papers [6], [7], [8], [9], one can see that the late time radiative behaviour of a neutral scalar field for asymptotically flat (R-N) and asymptotically (anti)-de-Sitter (RNdS and RNAdS) black hole space-times is essentially different: in the first case the inverse power-law tails are dominating, while in the second it is an exponential decay. This decay is oscillatory for RNAdS, and for RNdS when a scalar field strongly coupled to curvature.

When collapsing a charged matter, a charged black hole forms. Thus the evolution of a charged scalar field outside the R-N black hole is a most relevant. The late time behaviour of a charged scalar field was considered by S.Hod and T.Piran [10]. There was shown that in the radiative tails the neutral perturbations decay faster than the charged ones and therefore dominate at very late times. In addition, while at timelike and null infinity inverse power-law tails appear, along the future black hole event horizon, an oscillatory behaviour accompanies this tail.

Thus there is a quite clear picture of asymptotic behaviour of the radiation of charged perturbations, while its behaviour during the stage of quasinormal ringing is lacking. This motivated us to study the behaviour of a complex (charged) scalar field during the quasinormal ringing through the computing of its resonant characteristic frequencies for R-N and R-N AdS black holes. In Sec.II we shall compute the quasinormal frequencies of the R-N black hole for different multipole numbers l , and in Sec.III the case of the R-N-AdS black hole is considered. We have found that the modes of charged and neutral perturbations of the nearly extremal R-N black hole have the same damping times. The possible connection of this fact with the critical collapse is discussed in Sec.IV.

2 Reissner-Nordsrom black hole

We shall consider the evolution of the charged scalar perturbations field in the background of the Reissner-Nordstrom metric:

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega_2^2, \quad (1)$$

where $f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$. The wave equation of the complex scalar field has the form:

$$\phi_{;ab}g^{ab} - ieA_ag^{ab}(2\phi_{;b} - ieA_b\phi) - ieA_{a;b}g^{ab}\phi = 0, \quad (2)$$

here the electromagnetic potential $A_t = C - \frac{Q}{r}$, C is constant. After representation of the charged scalar field into spherical harmonics and some algebra the equation of motion takes the form [10]:

$$\psi_{,tt} + 2ie\frac{Q}{r}\psi_{,t} - \psi_{,r^*r^*} + V\psi = 0, \quad (3)$$

where

$$V = f(r) \left(\frac{l(l+1)}{r^2} + \frac{2M}{r^3} - \frac{2Q^2}{r^4} \right) - e^2 \frac{Q^2}{r^2}, \quad (4)$$

and $\psi = \psi(r)e^{-i\omega t}$. One can compute the quasinormal frequencies stipulated by the above potential by using the third order WKB formula of S.Iyer and C.Will [11]:

$$\frac{iQ_0}{\sqrt{2Q_0''}} - \Lambda(n) - \Omega(n) = n + \frac{1}{2}, \quad (5)$$

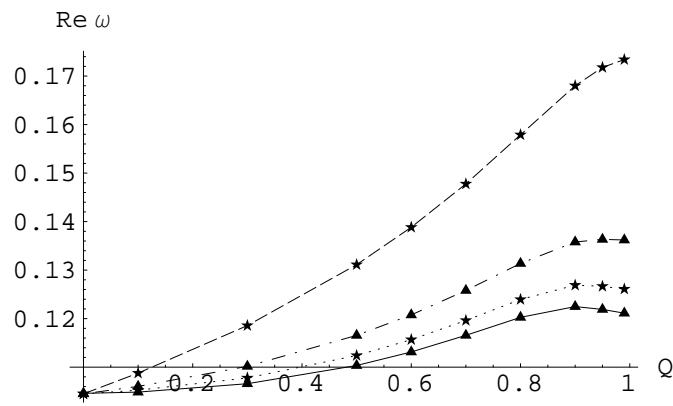


Figure 1: Real part of ω , $l = 0$, $n = 0$ for $e = 0, 0.01, 0.03, 0.1$ from the bottom to the top (R-N BH).

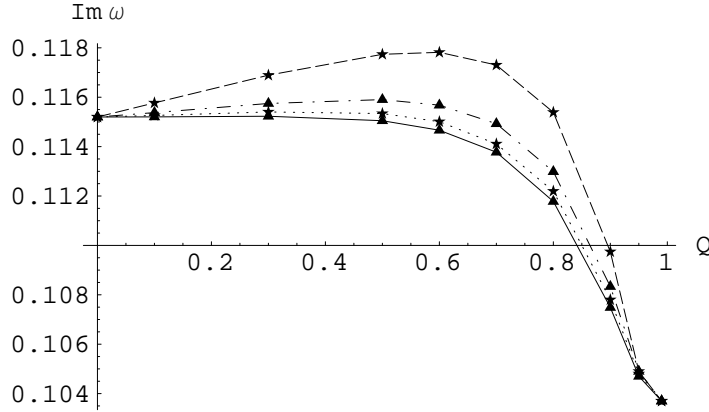


Figure 2: Imaginary part of ω , $l = 0$, $n = 0$ for $e = 0, 0.01, 0.03, 0.1$ from the bottom to the top (R-N BH).

where $\Lambda(n)$, $\Omega(n)$ are second and third order WKB correction terms depending on the potential V and its derivatives in the maximum. Here $Q = -V + \omega^2 - 2\frac{eQ}{r}\omega$. Since Q depends on ω , the procedure of finding of the QN frequencies is the following: one fixes all the parameter of the QN frequency, namely, the multipole index l , the overtone number n , the black hole mass and charge M and Q , and e ; then one finds the value of r at which V attains a maximum as a numerical function of ω and substituting it into the formula (5) one finds, with the trial and error way, ω which satisfies the equation (5).

$Re\omega$ for a neutral scalar field is known to grow up to some maximum with increasing of charge and then fall down. The maximum is posed close to the extremal values of Q . From the Fig.1 one can see that the real frequency of oscillation grows with increasing of e , and, the more e , the closer above maximum to the extremal value of Q .

In addition, and this is the most interesting feature of charged scalar QN spectrum, the imaginary part of a given "charged mode" approaches the neutral one in the limit of the extremal black hole. Certainly the modes coincide only within the accuracy of the

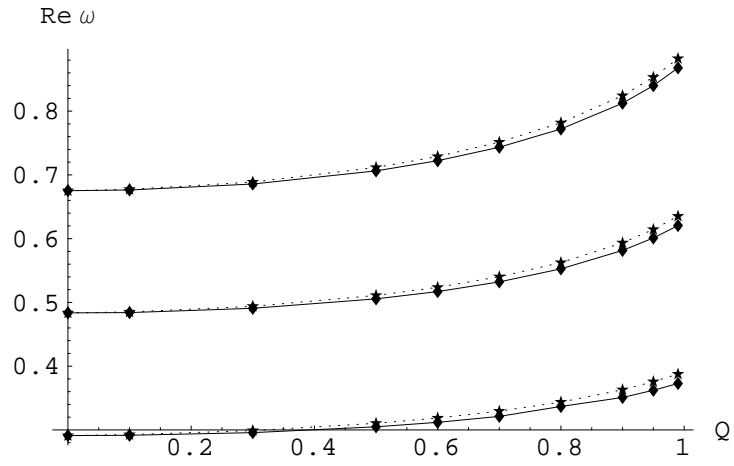


Figure 3: Real part of ω , $l = 1, 2, 3$, $n = 0$, $e = 0$ and $e = 0.03$ (R-N BH).

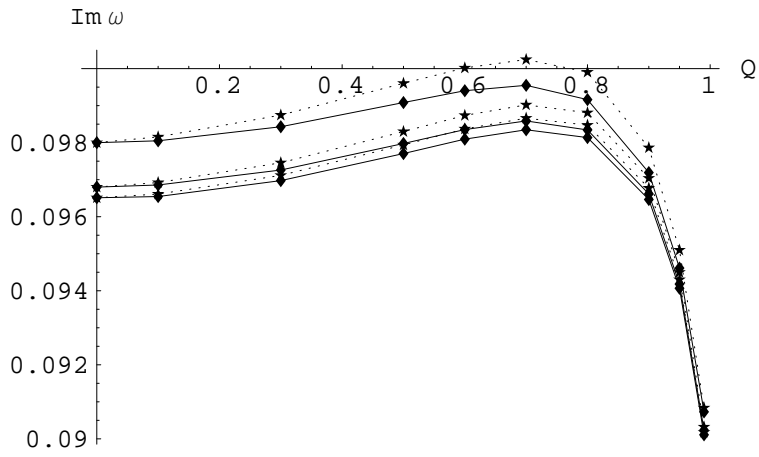


Figure 4: Imaginary part of ω , $l = 1, 2, 3$, $n = 0$, $e = 0$ and $e = 0.03$ (R-N BH).

3 Reissner-Nordsrom-Anti-de-Sitter black hole

The Reissner-Anti-de-Sitter metric has the form

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2d\Omega^2, \quad (6)$$

where

$$f(r) = 1 - \frac{r_+}{r} - \frac{r_+^3}{rR^2} - \frac{Q^2}{rr_+} + \frac{Q^2}{r^2} + \frac{r^2}{R^2}. \quad (7)$$

Quasinormal oscillations associated with the decay of the charged scalar field in the background of RNAdS are governed by the wave equation (3), which can be transformed to the form

$$f(r)\frac{d^2\psi(r)}{dr^2} + (f'(r) - 2i\omega)\frac{d\psi(r)}{dr} - V(r)\psi(r) = 0. \quad (8)$$

Here the $V(r)$ is, again, a frequency dependent potential, determined by the formula:

$$V(r) = \frac{f'(r)}{r} + \frac{l(l+1)}{r^2} + \frac{1}{f(r)} \left(2\frac{eQ}{r}\omega - e^2\frac{Q^2}{r^2} \right) \quad (9)$$

By rescaling of r we can put $R = 1$. The effective potential is infinite at spatial infinity. Thus the wave function is considered to vanish at infinity and satisfies the purely ingoing wave condition at the black hole horizon. Then one can compute the quasinormal frequencies stipulated by the above potential following the procedure of G.Horowitz and V.Habeny [3]. The main point of that approach is to expand the solution to the wave equation (8) around $x_+ = \frac{1}{r_+}$ ($x = 1/r$):

$$\psi(x) = \sum_{n=0}^{\infty} a_n(\omega)(x - x_+)^n \quad (10)$$

and to find the roots of the equation $\psi(x=0) = 0$ following from the boundary condition at infinity. In fact, one has to truncate the above sum at some large $n = N$ and check that for greater n the roots converge.

While the quasinormal modes of an asymptotically flat black hole are proportional to its mass, those of an asymptotically anti-de-Sitter black hole depend upon the radius of a black hole. For large (r_+ is much greater than the anti-de-Sitter radius R) and intermediate Schwarzschild-Anti-de-Sitter black holes, both ω_{Re} and ω_{Im} are proportional to the black hole temperature. For small black holes $r_+ \ll R$ this linearity breaks and in the limit $r_+ \rightarrow 0$ the QNM approaches the pure Anti-de-Sitter modes [3], [17]. Since it is a large black hole which is of direct interest of AdS/CFT correspondence, we shall restrict ourselves to this black hole regime.

From the Fig.5,6 one can see that ω_{Re} and ω_{Im} grow with increasing of the charge conjugation e , i.e. the real oscillation frequency is more for charged perturbations than for a neutral ones and the damping time of a given mode is more for neutral perturbations. Yet we managed to compute only the lowly charged case, due to the two difficulties. First, when Q and e grow, the number of terms in the truncated sum representing the

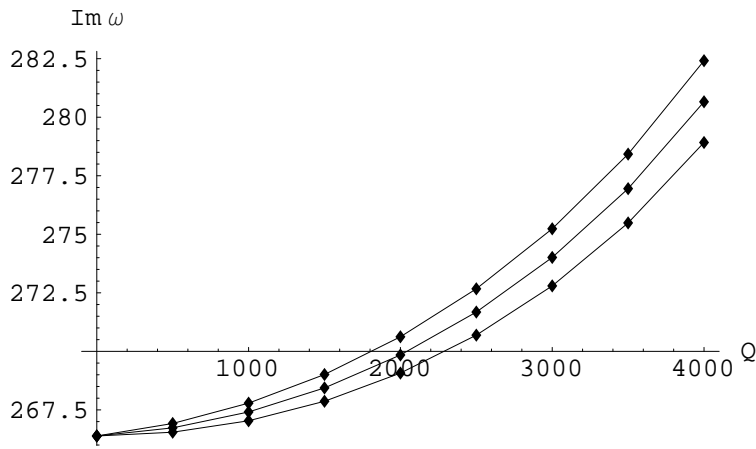


Figure 5: Imaginary part of ω , $l = 0$, $n = 0$ for $e = 0$, $e = 5 \cdot 10^{-5}$, and $e = 10^{-4}$ from the bottom to the top (R-NAdS BH).

wave function ψ increases: one has to sum over $N \sim 10^3$ and more, and thus one has to guess new modes though the trial and error way. At the same time, the minimums of the truncated sum corresponding to the quasinormal frequencies are a most narrow in the ω plane and one has to guess a lot of figures in the quasinormal frequency in order to catch the above minimum. Thus for highly charged case one has to resort to another method of calculations of QN modes.

4 Discussion

We have learnt here that the damping time of the quasinormal oscillations associated with a charged scalar field in the background of the Reissner-Nordstrom black hole is less than that of a neutral one. Thus that is the neutral perturbations which will dominate at later stages of quasinormal ringing. Yet, we know at late times the charged perturbations are dominating, and one could expect, possibly, that, the same sort of perturbations must be dominating in the earlier stages of radiation. The logic of the process however, is different. As was shown in [10] the late time behaviour of the charged scalar field is entirely determined by the flat space-time effects, while that of the neutral perturbations is depend on the relation between the "tortoise" r^* coordinate and r , i.e. by the space-time curvature. In other words, the radiative tail of the charged field arises due to the backscattering of this field off the electromagnetic potential far away from the black hole, while in the case of the neutral fields it is the effects of gravitation near the black hole (curvature effects). In this context it seems natural that in the earlier periods of radiation (quasinormal ringing) the curvature effects are dominating, and the neutral perturbations will damp slower.

Being interesting whether the charged perturbations decay faster than the neutral ones during the quasinormal ringing for *all* black holes we checked that this is the case for a non-extremal dilaton black hole (the corresponding data are available in electronic form from the author). Unfortunately in the near extremal regime the corresponding

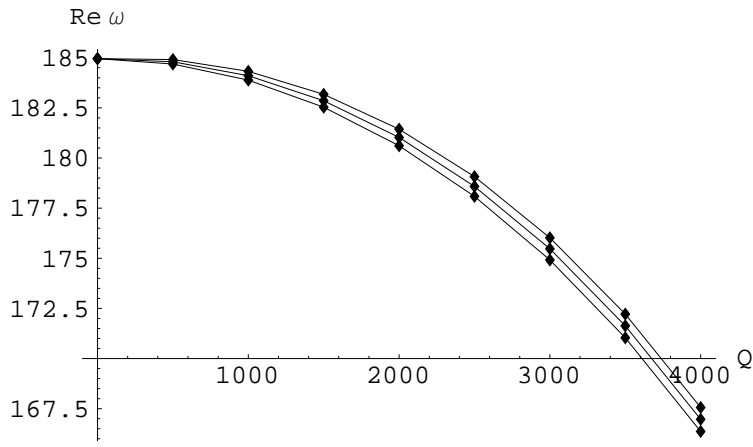


Figure 6: Real part of ω , $l = 0$, $n = 0$ for $e = 0$, $e = 5 \cdot 10^{-5}$, and $e = 10^{-4}$ from the bottom to the top (R-NAdS BH).

effective potential is broadening and we can not trust our WKB results. Gravitational QN frequencies of dilaton black holes were considered in [12] and [13].

Another interesting point of this study is the coincidence of the imaginary parts of ω for charged and neutral perturbations for the nearly extremal black hole. It leaps to the eyes at once that since the universal index appearing in the phenomena of critical collapse β equals 0.37 both for charged [14] and neutral [15] scalar fields, then there is a connection between the behavior of the quasinormal spectrum of nearly extremal black holes and the critical exponent for a given black hole. Yet the latter conjecture may be too strong, and the black hole quasinormal modes may be related to the critical exponent in some specific space-time geometries [3],[16].

In this connection it is interesting to remind that the nearly extremal R-N black hole is effectively described by the AdS_2 black hole after spherically symmetric dimensional reduction [18]. For such a reduced nearly extremal black hole an exact relation between the quasinormal modes and the universal index of critical collapse is obtained in [16]. Nevertheless an explanation of the coincidence of damping times for charged and neutral perturbations of the nearly extremal R-N black hole from physical point of view is lacking.

Whether the damping times of charged and neutral perturbations in the nearly extremal limit will coincide for RNAdS BH, and, for an asymptotically non-flat black hole in general, is a question for further investigation.

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